The Study of the Spatial Coherence of Surface Waves by the Nonlinear Green-Naghdi Model in Deep Water

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LONG-TERM GOAL

The goal is to identify the role of nonlinear wave interaction in the spatial coherence of ocean waves. For this purpose, a numerical tool based on Green-Naghdi model to simulate short-crested sea-state is being developed.

OBJECTIVES

The objectives are to develop higher-level Green-Naghdi model, to provide oceanographers and/or ocean engineers a new, numerical nonlinear-wave model, and to simulate fully-nonlinear interaction in short-crested random ocean surface efficiently. Emphasis will be made on the optimization between accuracy and computational effort by adjusting the `Level' of the model.

APPROACH

The irrotational version of the GN model, or the IGN model (see Kim et al., 1999), will be used to model the nonlinear evolution of ocean waves. The accuracy of the model can be controlled by the 'Level' of the model, which is defined as the number of interpolation functions in the vertical direction. As the Level goes up, the model describes the physics more accurately with a penalty of higher computational effort. The Level will be chosen such that simulations can be performed with minimal effort and redundancy in accuracy. The optimized model will be discretized by pseudo-spectral method on the horizontal plane.

WORK COMPLETED

The first year of the project was devoted to the derivation, numerical implementation, and the validation of the IGN model. The general derivation of the theory is submitted to the Journal of Engineering Mathematics and is under the final stage of review. The theory is also applied to the hydroelastic problem of a mat-type structure and several publications are produced on this subject (also related to an NSF project). As numerical implementation, pseudo-spectral codes for two- and three-dimensional problem have been developed. Validation of the theory and the numerical code was made from the comparisons with known exact solutions and experiments. The agreement is good. The validated code was applied to simulate long-crested random seas. The occurrence of extreme wave height, one of the features of the coherent structure of waves we are interested in, has been successfully simulated using the two-dimensional code. The other feature of the wave coherence, the crest-length problem, will be addressed after the three-dimensional code is validated.

RESULTS

Convergence of the IGN model

The convergence and the accuracy of the IGN model was tested by performing steady-state analysis of a Stokes wave. Also checked was the comparison with the original GN model. Fig. 1(a) shows the convergence of the IGN model to the semi-analytic result of Longuett-Higgins (1978). Fig. 1(b) shows the comparison of the present results with the GN model, taken from Webster & Kim (1990). It can be seen that the IGN solution converges to the analytic solution and the convergence rate is faster than the GN model after the vertical interpolation is optimized (see Kim & Ertekin, 1999 for more details).

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(a)   (b)
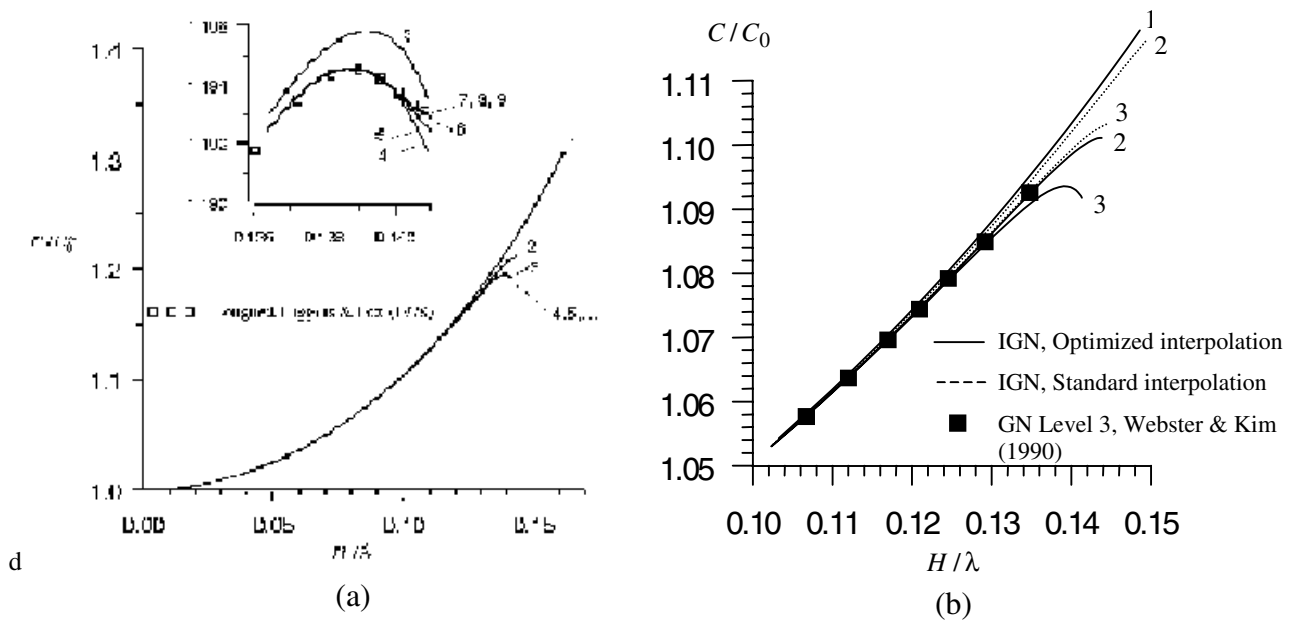
Fig. 1 Nonlinear dispersion of Stokes waves in deep water predicted by the IGN model. The numbers on the results denote the Level: (a) Comparison with the analytic solution of Longuett-Higgins & Fox (1978), (b) Comparison with the solution of the original GN equation.

Comparison with experiments

The field experiment of Su (1982) is simulated and compared with the measurements. Su's experiment covers two important features of nonlinear interaction among the waves, namely, the type I and type II instability. In this numerical test (in two dimensions), the capability of the new model to capture the type I instability is tested. Fig. 2 shows the wave elevation at eight different locations of wave gauges obtained from Level 3 IGN model and the measurements of Su (1982) when the frequency of the carrier waves is 1.34 Hz and steepness, a_0k_0, is 0.22 (Fig 4 in Su's paper). There are 10 waves in the wave train, which give 20 peaks in the wave gauge signal since the group velocity is two times slower than the phase speed. To consider the energy dissipation due to wave breaking, smoothing of the surface elevation and potential is made using an ideal filter described in Dommermuth and Yue (1987). The agreement is good.

IGN Level 3   Experiment (Su, 1982)

$\frac{2\pi\zeta}{\lambda}$
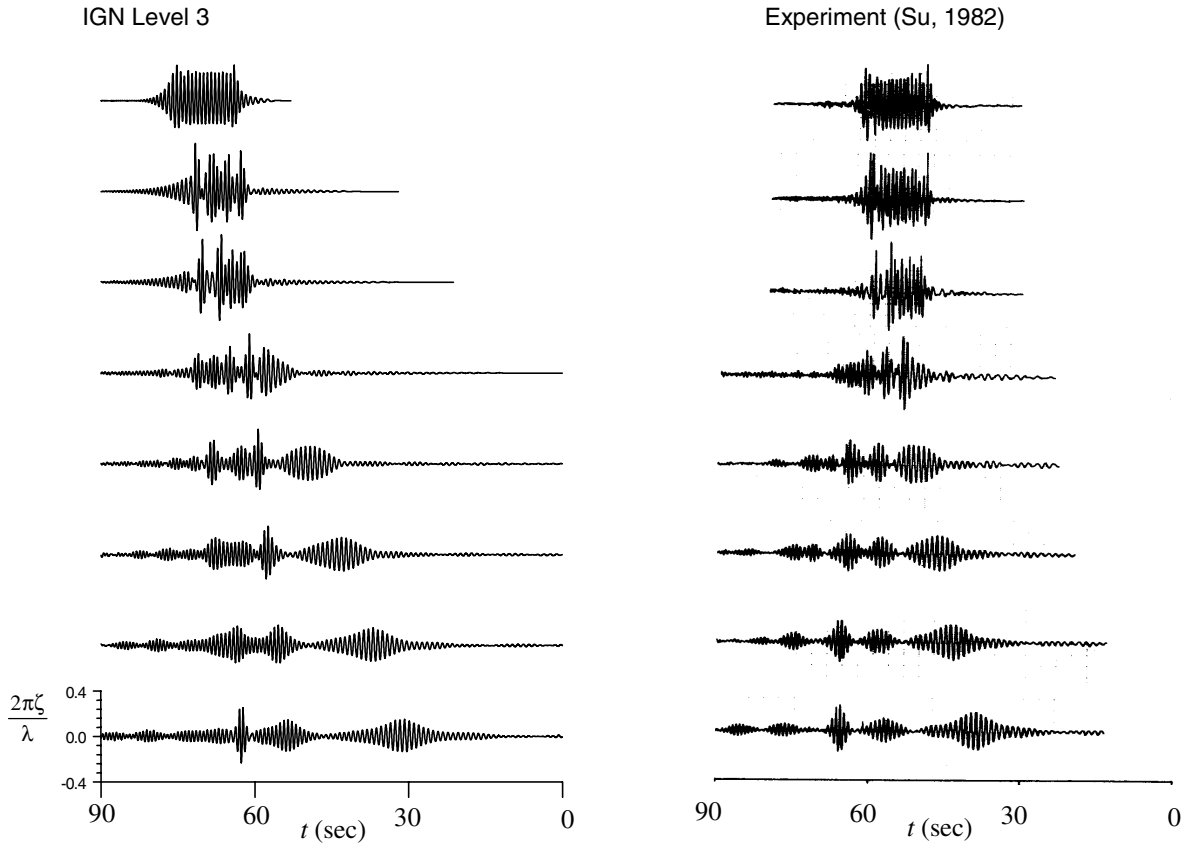
Fig. 2 Computed versus experimentally measured free-surface elevations at eight different wave gauges. Numbers between the figures denote the distance from the wavemaker.

Long-crested random seas

The time evolution of the long-crested random sea surface is simulated using the linear and nonlinear IGN models. Fig. 3 shows the time history of the wave elevation at three different locations. At $t \approx 22T_p$, a large wave, with a strong crest-trough asymmetry and extreme wave height, $H = 2.1H_{1/3}$, was observed at Gauge 3, denoted by G3 in the figure. The linear model could not predict such an extreme wave height. It was also observed that the wave profile around this
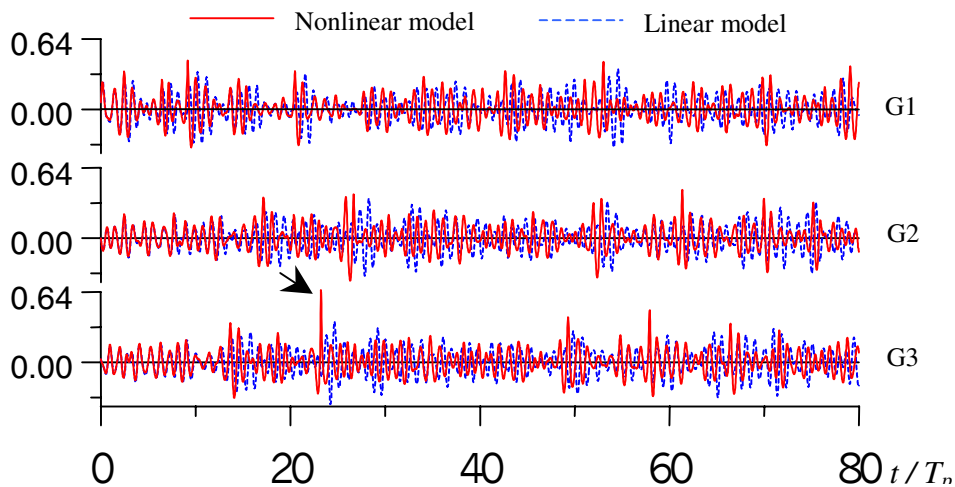

Fig. 3 Time history of surface elevation at three different numerical wave gauges. The IGN Level 2 model is used.

extreme wave height is quite similar to that obtained from the simulation of type I instability of the Stokes wave train (see Kim & Ertekin, 1999 for more details). The generation of extreme wave height due to type I instability was also exhibited by the 'rogue wave' solution of NLS equation (Osborne, 1999). We are performing validation tests of the 'rogue wave' solution using the IGN model and the results will be reported soon.

Validation of three-dimensional model

The numerical model of the three-dimensional IGN equation is under the validation process. Fig.4(a) shows the propagation of Stokes waves in skew direction. The conservation of the wave form and mechanical energy were checked and determined to be satisfactory. Fig. 4(b) shows the amplification of type II instability three periods after the seeding of initial instability. The initial instability mode was given based on the theoretical study of Stiassnie & Shemer (1987).

(a)   (b)
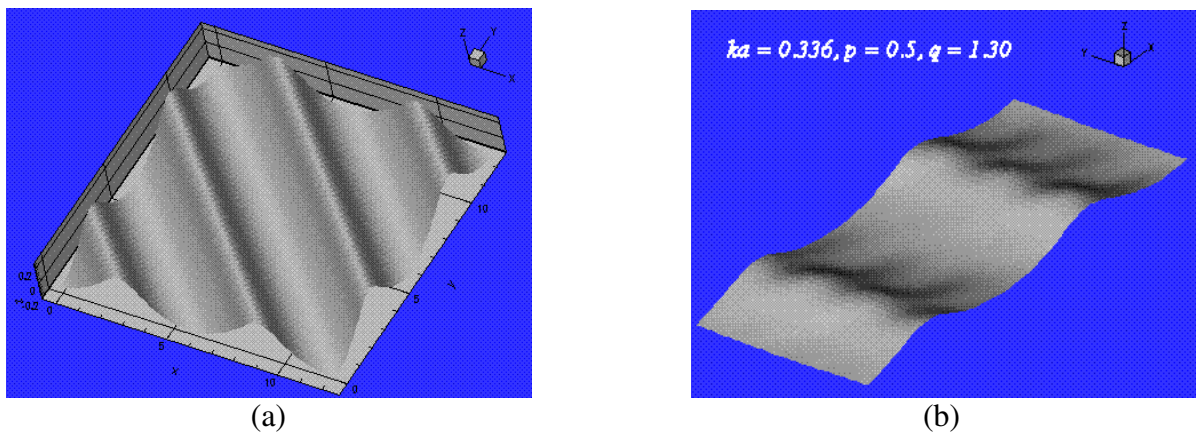
Fig. 4 Snapshot of surface elevation.
(a) Stokes wave; and (b) Stokes wave with Type II instability

IMPACT/APPLICATION

A new numerical model for nonlinear evolution of ocean waves is being developed. The new model will provide the wave environmental input for designing very large floating structures such as the proposed Mobile Offshore Base (MOB).

TRANSITIONS

The shallow water version of the GN theory is applied to the hydroelastic problem of mat-type structures. The developed numerical code, GNplate, is being used in NSF project: Grant No.: BES-9532037, Co-PIs: H.R. Riggs and R.C. Ertekin, to evaluate the performance of floating runways.

RELATED PROJECTS

NSF Grant No.: BES-9532037

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